Transmit Beamforming for Acoustic OFDM

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Hey, I'm talking to you!

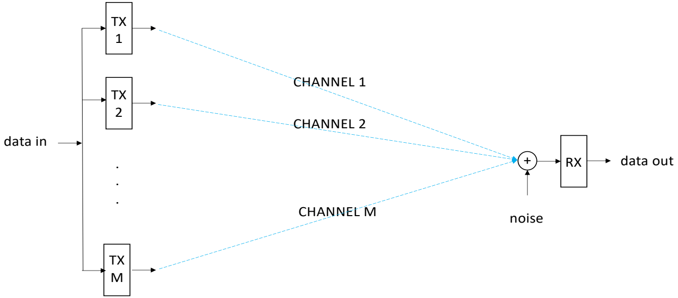
Transmit beamforming

- **Why**:
  - efficient use of power
  - avoidance of unintended listeners
  - spatial division of multiple users
- **How**: Assign a weight to each transmit element so that their signals add constructively at the receiver.
- **Problem**: Weights depend on the channel. Downlink channel must be inferred from uplink. Estimate is noisy and delayed.
- What has been done: A lot in radio, a little in underwater acoustic systems (adaptive modulation, time reversal).

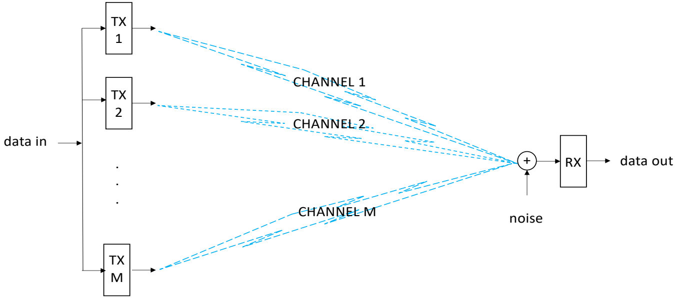
OFDM

- bandwidth scalability
- ease of FFT processing
- possibility of differentially coherent detection
- proven methods for acoustic Doppler compensation
- ideal platform for broadband beamforming

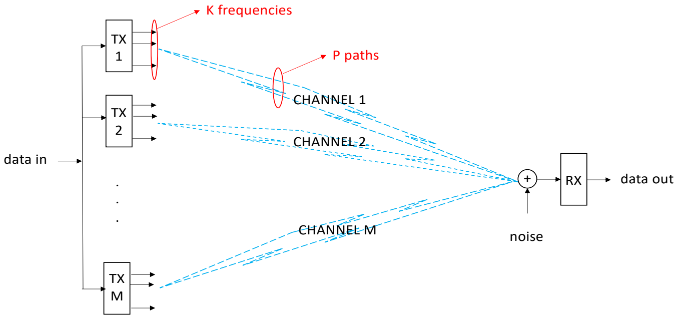
Basic setup: Multiple transmitters



Basic setup: Multiple paths



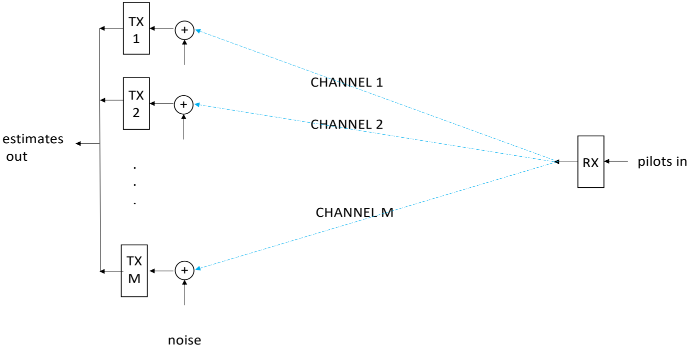
Basic setup: Multiple frequencies



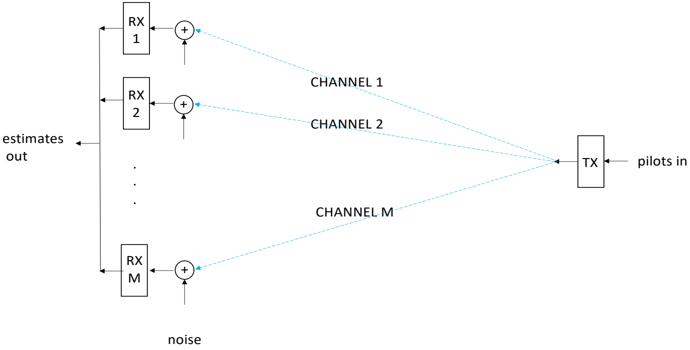
Optimal beamforming (sorry, no pretty pictures)

- $H_k^m = \sum_p h_p^m \gamma_p^m(f_k) e^{-j2\pi f_k \tau_p^m}$: channel transfer function on the k-th carrier, m-th element
- w_k^m: beamformer weight on the k-th carrier, m-th element
- $y_k = d_k \sum_m w_k^m H_k^m + z_k = d_k \mathbf{w}_k^T \mathbf{H}_k + z_k$: signal received on the k-th carrier
- Maximum SNR: $\mathbf{w}_k \sim \mathbf{H}_k^*$
- Normalization: $\frac{1}{K} \sum_k \mathbf{w}_k^H \mathbf{w}_k = 1$
  no extra power expenditure on account of beamforming

Channel estimation via feedback (uplink)



Channel estimation via feedback (uplink)



Channel estimation

- Downink channel has to be estimated from the uplink pilot.
- If $\mathbf{H}^{dn} = \mathbf{H}^{up}$, the only problem is noise.
- During the time it takes to close the feedback loop ($\sim 2d/c$), the system geometry could change (ever so slightly), hence $\mathbf{H}^{dn} \neq \mathbf{H}^{up}$.
- $x_k^m = H_k^{m,up} + z_k^{m,up}$: received pilot signals
- $\hat{\mathbf{H}}^{dn} = \mathcal{L}(\mathbf{X}) = \mathbf{H}^{up} + \alpha \mathbf{Z}^{up}$: estimated channel
- $\alpha = 0$: noiseless estimate
  $\alpha = \sqrt{\frac{L}{K}}$, $L = \lceil BT_{mp} \rceil$: LS estimation in the IR domain
  $\alpha = 1$: time reversal (TR)

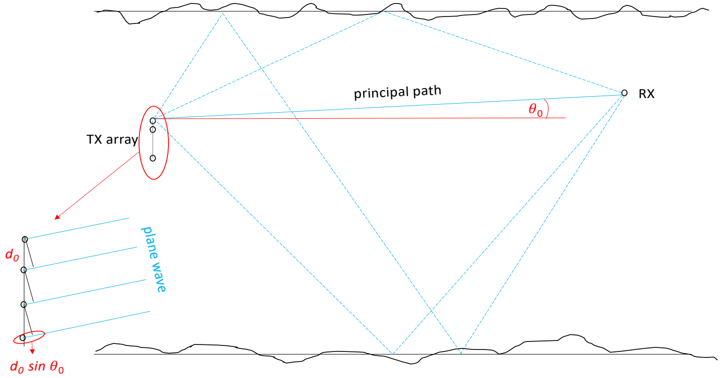
Beamforming with a channel estimate

- $y_k = d_k \sum_m w_k^m H_k^{m,dn} + z_k^{dn} = d_k \mathbf{w}_k^T \mathbf{H}_k^{dn} + z_k^{dn}$
- Noises are characterized by σ_{dn}^2, σ_{up}^2.
- $\mathbf{W} \sim \hat{\mathbf{H}}^{dn*}$: Beamformer weights are still determined according to the maximum SNR rule, but a channel estimate is used instead of the unknown true value.
- Case studies: perfect channel knowledge, delayed channel (noiseless estimate), TR, IR; no adjustment ($w_k^m = \frac{1}{\sqrt{M}}$).

How good will the channel estimate be?

- The time to close the feedback loop allows for the following changes to occur:
  (i) small-scale fading coefficients γ_p^m
  (ii) system geometry (e.g. rx drifts at speed v in direction θ_R).
- Neither is fully predictable.
- Both can cause a significant change in the channel response.
- **Grand question**: Is there some feature of the channel that changes slowly enough that it can withstand the feedback delay, yet be exploited to formulate an efficient beamformer?

Beamforming in the principal path's direction



Beamforming in the principal path's direction

- Principal path is stable (no surface interaction).
- Drifting will cause tx-rx positioning to change by a few meters over a few seconds, but this change is negligible compared to a transmission distance on the order of kilometers. The change in the principal path's angle of arrival is thus expected to be negligible.
- Note: receiver must still compensate for the Doppler shift (on either side of the link).

Beamforming: Classical approach (array processing)

- Plane wave propagation, equally-spaced array elements (d_0), signal coming from direction θ_0:
  $\tau_0^m = \tau_0^0 + m\Delta\tau_0$, $m = 0, \ldots, M-1$
  $\Delta\tau_0 = \frac{d_0}{c}\sin\theta_0$
- If the signal is properly synchronized, $\tau_0^0 = 0$.
- For a narrowband signal of frequency f_0, beamformer weights are $w_0^m = \frac{1}{\sqrt{M}} e^{j2\pi f_0 \tau_0^m} \sim e^{j2\pi m f_0 \frac{d_0}{c}\sin\theta_0}$ (or $e^{j2\pi m \frac{d_0}{\lambda_0}\sin\theta_0}$)
- Acoustic communication signal is not narrowband, so beamforming weights are set for each frequency f_k,
  $w_k^m = e^{j2\pi m f_k \frac{d_0}{c}\sin\theta_0}$

Channel-based vs. angle-based beamforming

- $H_k^m = \sum_p h_p^m \gamma_p^m(f_k) e^{-j2\pi f_k \tau_p^m} = \sum_p \underbrace{h_p^m \gamma_p^m(f_k) e^{-j2\pi f_k \tau_p^0}}_{\tilde{h}_p^m(f_k)} e^{-j2\pi m f_k \Delta\tau_p}$
- $w_k^m \sim H_k^{m*} = \underbrace{\tilde{h}_0^{m*}(f_k)}_{h_0 e^{-j2\pi f_k \tau_0^0}} \underbrace{e^{j2\pi m f_k \frac{d_0}{c} \sin\theta_0}}_{steering\ \ to\ \ \theta_0} + \sum_{p\neq 0} \underbrace{\tilde{h}_p^{m*}(f_k)}_{?} \underbrace{e^{j2\pi m f_k \frac{d_0}{c} \sin\theta_p}}_{steering\ \ to\ \ \theta_p}$
- Optimal beamforming: match the phase and gain of every path and array element.
- Beamforming in the principal direction: treat all other paths as nuisance (give up on predicting small-scale fading coefficients on the downlink), and just steer the beam in direction θ_0.

Estimating the principal angle of arrival

- Let $w_k^m(\theta) = \frac{1}{\sqrt{M}} e^{j2\pi m f_k \frac{d}{c} \sin\theta}$
- Beamforming on the k-th carrier of the uplink signal would yield $\sum_m w_k^m(\theta) x_k^m = \mathbf{w}_k^T(\theta)\mathbf{x_k}$.
- Total power after beamforming is $S(\theta) = \sum_k |\mathbf{w}_k^T(\theta)\mathbf{x_k}|^2$
- Principal angle is estimated as that angle for which the power is maximized: $\hat{\theta}_0 = \arg\max_\theta S(\theta)$
- Beamformer weight are set to $w_k^m = w_k^m(\hat{\theta}_0)$.

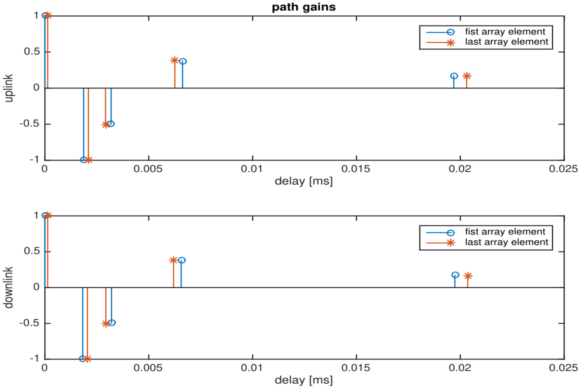
Notes on narrowband beamforming

- Narrowband beamforming rests on the assumption that the signal bandwidth is much smaller than the center frequency.
- The weights assigned to different transmit elements are the same for all signal frequencies.
- If the weights are evaluated for f_0, then $w_k^m = w_0^m$.
- For angle-based narrowband beamforming, $\hat{\theta}_0 = \arg\max_\theta \sum_k |\mathbf{w}_0^T(\theta)\mathbf{x_k}|^2$, $w_k^m = w_0^m = \frac{1}{\sqrt{M}} e^{j2\pi m f_0 \frac{d}{c} \sin\hat{\theta}_0}$
- Question: How much is the performance degraded under narrowband assumption over underwater acoustic channels?

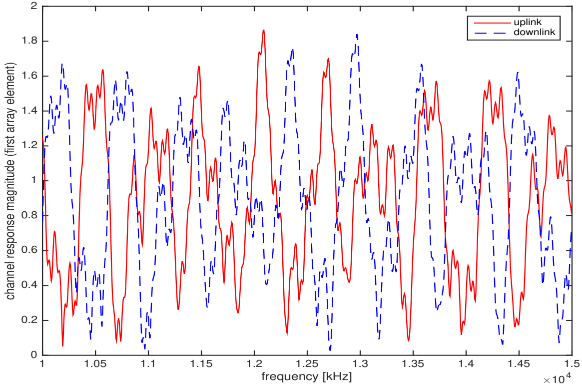
Numerical illustration

- System geometry:
  $d = 1$ km, $h = 100$ m, $h_R = 20$ m, $h_T = 70$ m (top)
  $v = 0.5$ m/s, $\theta_R = 45^\circ \Rightarrow d$ increases, h_R decreases by 0.47 m between uplink/downlink time
- Frequency occupancy: $f_0 = 10$ kHz, $B = 5$ kHz
- Transmit array: $M = 12$, $d_0 = 0.345$ m ($2.3\lambda_{max}$ or $3.45\lambda_{min}$)
- h_p^m, τ_p^m calculated from system geometry; c=1500 m/s in water, 1300 m/s in bottom
- Small-scale fading: independent across array elements; $\sigma_s, \sigma_b \sim \lambda_0$, $B_{\delta_p} = 10^{-4}$ Hz
- Signal: $K = 1024$, differential QPSK
- Noise: $\sigma_{up}^2 = \sigma_{dn}^2 = \sigma^2$, SNR=$\frac{1}{\sigma^2}$ with $\frac{1}{MK}\sum_m \sum_k |H_k^{m,up}|^2 = 1$

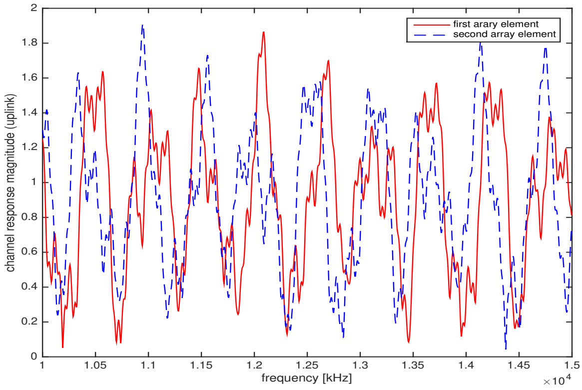
Multipath structure



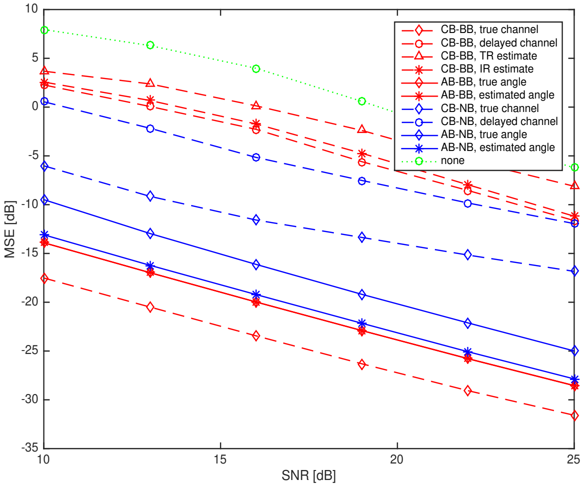
Frequency response of the channel: Uplink vs. downlink



Frequency response of the channel: Across the array



Performance results



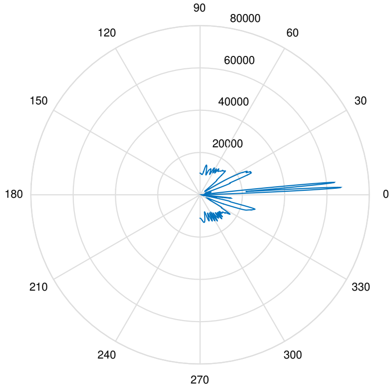
CB/AB=channel based/angle based, BB/NB=broadband/narrowband

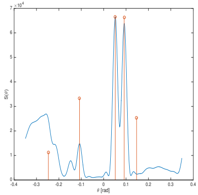
delayed channel = uplink channel, noiseless

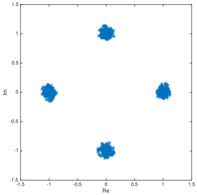
Inside angle-based beamforming: $S(\theta)$

SNR=20 dB

$\theta_0 = 0.05$ rad (0.05-0.0537 up; 0.0504-0.0542 down; 0.0524 est.)







Resolution and ambiguity limits

- $\frac{\lambda_{max}}{M\Delta_{max}} < d_0 \leq \frac{\lambda_{min}}{\Delta_{min}}$

  $$\Delta_{max,min} = \max, \min_{(p,q)} \{\sin\theta_p - \sin\theta_q\}$$

- LHS: Paths p and q are resolvable.
- RHS: There is no ambiguity as to whether the angle of path p is x degrees above or below the angle of path q.
- Note: include only those paths that are stable and not negligible in strength.

Conclusion

- When the channel varies over the time it takes to close the feedback, discrepancy between the uplink and the downlink is significant enough to rule out channel-based beamforming.
- A possible solution is angle-based beamforming in the direction of the principal path (that which has no surface interaction and is stable). Rx/tx motion over the uplink-downlink should not cause a significant change in the angle of a typical geometry.
- Next steps:
  angle tracking
  broadband null steering
  multiple users
  simultaneous tx/rx beamforming.